# A simple analysis of flow and heat transfer in railway tunnels

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A new method for the prediction of steady turbulent flow and heat transfer in the gap between a railway train and the wall of a long tunnel is proposed. The theory is based on the assumption of power laws for the velocity profiles adjacent the rough train structure and the smooth tunnel. The friction and heat transfer coefficients at the two surfaces are determined using correlations for imaginary rough and smooth ducts in place of the boundary layers. Preliminary calculations have been made for a realistic situation, and the velocity prediction is compared with that by a previous method. The limitations of the Reynolds Analogy in the train–tunnel thermal problem are examined quantitatively.

**Keywords:** *steady turbulent flow, heat transfer, railway tunnels*

## Introduction

When a train travels through a long tunnel, a significant amount of thermal energy may be transferred to the tunnel environment. This thermal energy is the result of dissipation caused by aerodynamic drag, mechanical resistances, and inefficiencies in the power unit. Proportions of this energy are transferred to the tunnel wall and the train structure, the remainder being expelled from the tunnel by the induced air flow. The energy transfer to the tunnel wall itself is of great importance, since after many years and successive train movements, it could cause an unacceptable rise in the rock temperature. This would then result in the need to restrict traffic levels and possibly necessitate the installation of a cooling system. Such considerations are of importance to the design of the proposed Channel Tunnel between the UK and France and in the 54 km long Seikan tunnel in Japan[1]. The heat transfer to the train structure, on the other hand, is generally beneficial since this ultimately results in thermal energy being removed out of the tunnel.

It is therefore vital to be able to make reliable predictions for those components of energy transfer at the design stage so that thermal problems in the tunnel environment may be eliminated. In the past, attempts have been made to calculate long-term tunnel temperatures but difficulties have arisen over the modelling of heat transfer to the train structure. Some workers chose to ignore this energy transfer, but Moron[2] and Pope and Woods[3] have demonstrated that the effect may be important.

Present prediction methods cannot be judged to be reliable, and there is a strong requirement for a more realistic heat transfer prediction method. Of particular interest are the heat transfer coefficients at both the train and tunnel surfaces, and how these are determined in the case of a train travelling through a long tunnel is the subject of the present enquiry. In the case of train–tunnel heat transfer, two special features arise. Firstly, there is a mixture of rough and smooth surfaces, and secondly, there is parallel relative motion between these surfaces.

In the following section, a new flow and heat transfer model is described and the procedure for determining the velocity field and heat transfer rates is outlined briefly.

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Manuscript received 21 July 1986 and accepted for publication on 12 January 1987

0142-727X/87/020119-05$3.00



## Theoretical considerations

In order to predict heat transfer rates to the surfaces, it is necessary to determine the velocity field and, hence, shear stress distribution in the gap between the 'rough' train surface and the 'smooth' tunnel wall. The velocity profile will be of type A or type B as shown in Fig 1, depending on the magnitude of the pressure gradient and the train velocity. In both cases, the shear stress distribution is linear across the gap is indicated. (In case A the drags on the tunnel and train surfaces are in the same direction, whereas in case B the drags are in the opposite directions.) Assuming fully developed flow, the momentum equation is simply

$$0=-\frac{dp}{dx}+\frac{d}{dy}(\tau) \quad (1)$$

when

$$L\left(\frac{dp}{dx}\right)=\tau_r \pm \tau_s \quad (2)$$

the positive and negative signs referring to cases A and B, respectively. The stresses shown in Fig 1 are of course those exerted on the fluid when the pressure gradient is positive.

With regard to the velocity distribution, it is assumed that power laws of the form

$$\left(\frac{u}{u_{max}}\right)=\left(\frac{y}{\delta}\right)^{1/m} \quad (3)$$

adequately describe the profiles in the rough and smooth domains adjacent the train and tunnel, respectively (see, for example, Schlichting[4]). Typically, $m=7$ for a smooth surface and $m=4$ for a rough wall, and these values have been chosen for the present analysis, each being assumed to be independent of Reynolds numbers. It is also tacitly assumed that the shear stress at the interface between the rough and smooth domains is zero, a practice frequently adopted in annular flows and other asymmetric situations.

The procedure to determine the velocity distribution across the gap is then to solve Eq (2) iteratively with Eq (3) for $m=7$ and 4 adjacent the tunnel and train surfaces, with $\delta$ in Eq (3) being the corresponding boundary layer thickness. Fig 2 shows how this matching of the separate velocity profiles is effected for case A. Of course, during the iteration, the smooth and rough wall boundary layer thicknesses vary. As part of the procedure, the two flows adjacent the train and tunnel are modelled as shown in Fig 3. That is, imaginary rough and smooth walled ducts, each twice the thickness of the respective boundary layers, are constructed. The purpose of these imaginary ducted flows is to enable friction (and heat transfer) correlations to be employed in the iteration. The additional relationships given in Table 1 may then be incorporated in the calculation for the velocity field across the whole gap. It is to be noted that the velocities for the train in (iv) in Table 1 are those relative to the train surface.

In the early development of the computing procedure, a train friction factor 0.012 was assumed. For the 'fully-rough' condition of the train surface, this friction factor is of course constant and independent of Reynolds number, as reference to the Moody diagram[5] will reveal. In the later stages of development of the program however, a friction factor correlation for a rough duct was introduced together with a general velocity correlation. In this way, it becomes possible, with the aid of a single velocity measurement adjacent to the rough surface, to iterate for the correct train friction factor and to obtain the equivalent roughness of the train surface as a by-product. With regard to this last refinement, the following two additional relationships were incorporated in the calculation:

$$\frac{1}{\sqrt{f}}=1.737\ln\left(\frac{d_e}{e}\right)+2.28 \quad (4)$$

and

$$u^+=2.5\ln\left(\frac{y}{e}\right)+8.5 \quad (5)$$

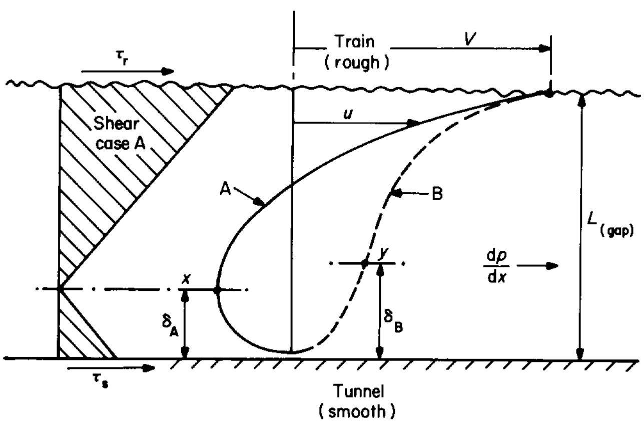


*Figure 1* Velocity and shear stress distributions

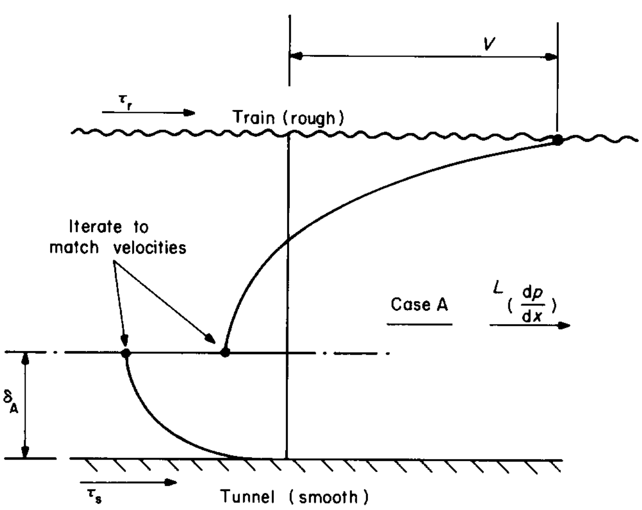


*Figure 2* Matching of velocity profiles in rough and smooth domains

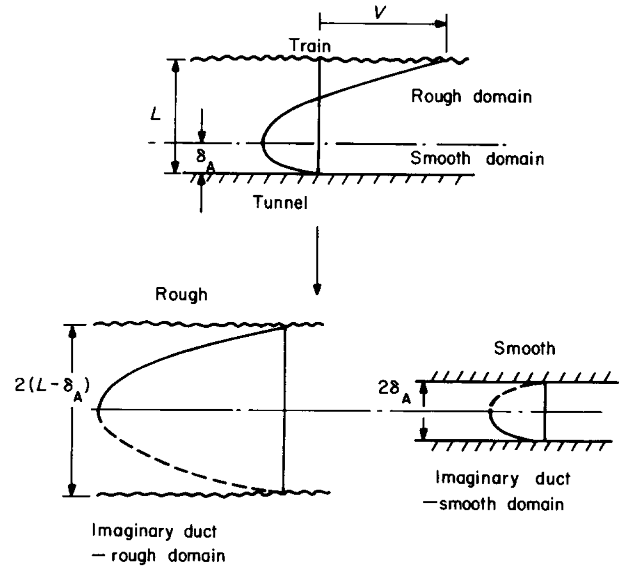


*Figure 3* Modelling of flow and heat transfer in the train surface and tunnel wall regions by means of imaginary ducted flows (case A)

**Table 1** Relationships incorporated in calculation of velocity field across whole gap

| Tunnel (smooth) | Train (rough) |
|---|---|
| (i) $f_s=\frac{2\tau_s}{\rho\bar{u}_s^2}=0.079\,Re_s^{-1/4}$ | $f_r=\frac{2\tau_r}{\rho\bar{u}_r^2}$ |
| (ii) $Re_s=\left(\frac{\bar{u}_s d_e}{\nu}\right)$ | $Re_r=\left(\frac{\bar{u}_r d_e}{\nu}\right)$ |
| (iii) $d_e=4\delta_A$ (case A) | $d_e=4(L-\delta_A)$ (case A) |
| (iv) $\left(\frac{\bar{u}_s}{u_{max}}\right)=0.817$ ($m=7$) | $\left(\frac{\bar{u}_r}{u_{max}}\right)=0.71$ ($m=4$) |

## Notation

| | |
|---|---|
| $d_e$ | Equivalent diameter defined in text |
| $e$ | Equivalent sand roughness |
| $f$ | Friction factor (Fanning) |
| $h$ | Heat transfer coefficient |
| $k$ | Constant |
| $L$ | Gap between train and tunnel surfaces |
| $m$ | Exponent |
| $Nu$ | Nusselt number |
| $P$ | Static pressure |
| $Pr$ | Prandtl number |
| $Q$ | Volume flow rate per unit width |
| $Re$ | Reynolds number |
| $St$ | Stanton number |
| $u$ | Local velocity |
| $u^+$ | Dimensionless velocity |
| $V$ | Train speed |
| $x$ | Distance along tunnel |
| $y$ | Wall distance |
| $y^+$ | Friction distance |
| $\delta$ | Boundary layer thickness |
| $\rho$ | Air density |
| $\tau$ | Shear stress |
| $\nu$ | Kinematic viscosity |

*Subscripts*

| | |
|---|---|
| A, B | Pertaining to profile types A and B |
| r | Rough |
| s | Smooth |

**Table 2** Convection heat transfer correlations for flows in the two imaginary ducts

| Tunnel (smooth) | Train (rough) |
|---|---|
| (i) $Nu_s = 0.02\,Re_s^{0.8}$ | $\frac{Nu_r}{Nu_{r,s}} = \left(\frac{f_r}{f_{r,s}}\right)^{0.5}$ where $Nu_{r,s} = 0.02\,Re_r^{0.8}$ |
| (ii) $d_e = 4\delta_A$ | $d_e = 4(1-\delta_A)$ (case A) |

where $f_{r,s}$ and $Nu_{r,s}$ are the friction and heat transfer coefficients for the train duct when assumed to have smooth surfaces.

These are well-known equations for ducts with rough surfaces. It can be seen that with $u$ known at a given wall distance $y$ the wall friction (and roughness) may then be determined in the calculation.

As for the friction, convection heat transfer correlations for the flows in the two imaginary ducts were employed in the thermal problem. Following the assumption concerning the friction at the interface between the rough and smooth domains, the heat flux there is also assumed to be zero. This then permits the use of correlations for symmetric heating in the imaginary ducts of width twice the respective boundary layer thicknesses. These correlations which employ the use of the equivalent diameter in the Nusselt and Reynolds numbers are listed in Table 2. (The relationships are simplified forms of the Dittus–Boelter equation with $Pr = 0.7$.) This type of correlation for heat transfer from a rough surface is reported in Kays and Crawford[6]. Strictly speaking, the exponent in the friction ratio is a function of Prandtl number, but the proposed expression is considered to be sufficiently accurate for present purposes.

The interdependence between skin friction and heat transfer through Reynolds Analogy is familiar* and the role of the relationship

$$St = \frac{f}{2} \tag{6}$$

in the tunnel–train situation requires careful examination. There are important restrictions in the use of the relationship, particularly with regard to the value of the Prandtl number, the distribution of shear and heat flux across the flow, and the degree of roughness of the surface. (With regard to this last effect, Reynolds Analogy by Eq (6) predicts the same percentage increase in heat transfer as for friction for a roughened surface!) Previously, Eq (6) has been used in this kind of problem without cognizance of these important restrictions being taken. In the present work, it is proposed to use a Reynolds Analogy factor, $2St/f$, to examine how closely the calculated friction and heat transfer coefficients conform to the theory. In this way a careful check may be made on the trends of the heat-transfer/friction ratio as the roughness increases.

To make a comparison of the flow prediction with other data, it is also convenient to consider the volume flow rate per unit width of the gap between train and tunnel. It is easy to show that for profiles of type A (see Fig 1),

$$Q_A = V - (V + u_{max,A})\frac{4}{5}(1-\delta_A) - \left(\frac{7}{8}u_{max,A} + V\right)\delta_A \tag{7}$$

and, for case B,

$$Q_B = u_{max,B}\left(\frac{3}{40}\delta_B + \frac{4}{5}\right) + \frac{V}{5}(1-\delta_B) \tag{8}$$

where $u_{max,A}$ and $u_{max,B}$ are the absolute velocities at the interface between the rough and smooth domains. The parameter $(Q/VL)$ may then be obtained for direct comparison with velocity data given by Nayak *et al*[7]. Furthermore, it is also straightforward to evaluate $(V^2k^2\rho/L\,dp/dx)^{0.5}$ with $k = 0.4$ for comparison with the data given in Fig 9 of Ref 7 for dimensionless pressure gradient and flow rate.

In the next section, a brief account of the results obtained so far by the new prediction method is given. Comparison with the results of Nayak *et al*[7] for the velocity is possible using arbitrarily selected conditions and dimensionless forms of the parameters.

## Results

As stated in the previous section a constant train friction factor equal to 0.012 was chosen in the preliminary calculations during the development of the program. In the 'fully-rough' region, of course, the friction factor depends on relative roughness only, so that a particular value of this parameter is now inferred. In Fig 5, a comparison is made between the velocity distribution of Nayak *et al*[7] for a value of $(Q/VL) = -0.588$ and that according to the present prediction for $(L\,dp/dx) = 5$ corresponding to $(Q/VL) = -0.575$.

Albeit that these values of the dimensionless flow rate are not exactly the same, the agreement is good, keeping in mind the completely different approaches used in the two studies. The dependence on dimensionless flow rate is also illustrated by the inclusion of the profile for $(Q/VL) = -0.062$, which corresponds to a pressure gradient $L(dp/dx) = 2.2$.

Further comparison with the results of the work of Ref 7 may be made with the parameter $(V^2k^2\rho/L\,dp/dx)^{0.5}$ and this has been done for the two cases shown in Fig 5. This comparison is made in Table 3. The agreement here is also good, particularly at the larger values of flow rate.

The program was also designed to output the two heat transfer coefficients. The train surface coefficient was invariably

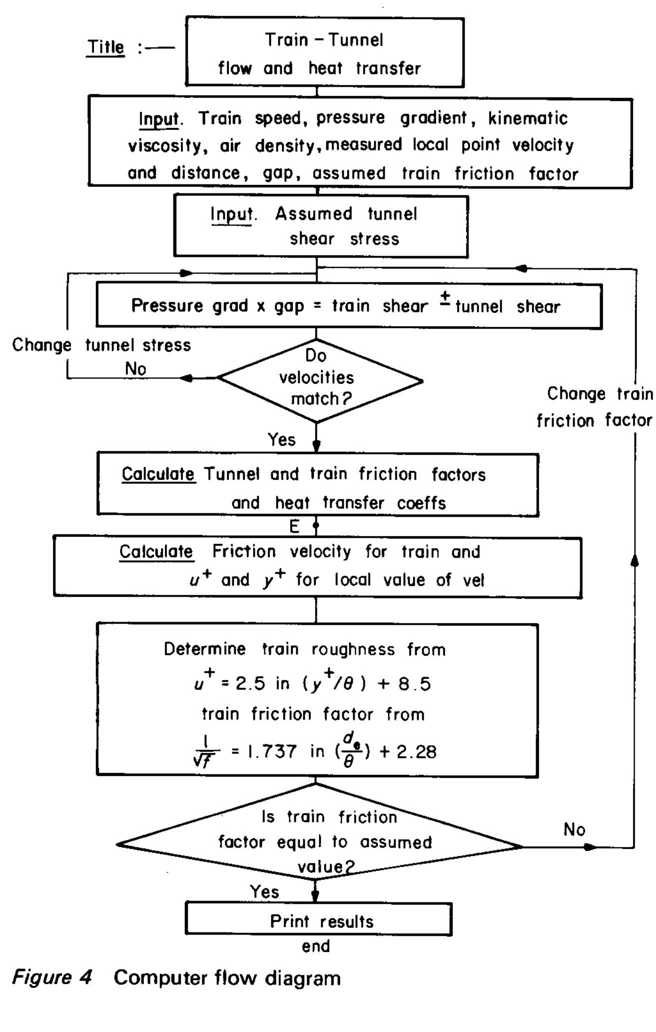


*Figure 4* Computer flow diagram

/ See standard heat transfer texts.

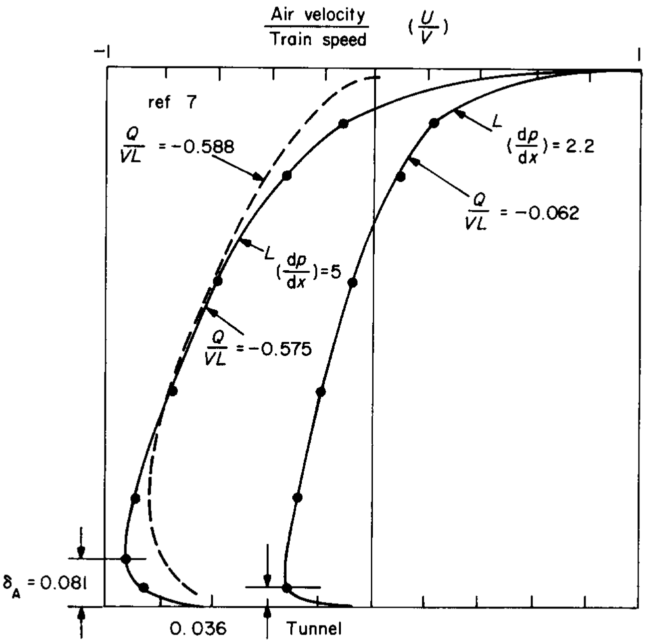


*Figure 5* Predicted velocity profiles for case A with $f$ (train) = 0.012

**Table 3** Comparison of present results with those of Ref 7 $\left(V^2k^2\rho/L\,\frac{dp}{dx}\right)^{0.5}$

| $\frac{Q}{VL}$ | Fig 9 of Ref 7 | Present study $f$(train) = 0.012 |
|---|---|---|
| −0.062 | ≈6.0 | 5.41 |
| −0.58 (average value) | ≈3.50 | 3.59 |

the larger one as anticipated, and typical values are given in Fig 6 which shows the effect of speed on heat transfer rate. Of course, train speed and pressure gradient are not independent parameters, and so Fig 6 is for the purpose of detecting trends and relative effects only. In such a complicated thermal situation, Fig 6 is useful, however, in assessing approximate magnitudes for the two heat transfer coefficients for engineering calculations.

In the previous section, mention was made of Reynolds Analogy, its limitations and its role in the present investigation. The Reynolds Analogy factor is shown plotted for the tunnel and train surfaces (using the imaginary duct model) versus train friction factor in Fig 7. Again, the pressure gradient cannot be chosen independently of friction factor but the message from Fig 7 is the very approximate nature of the relationship of Eq (6) in the train–tunnel situation. As expected, the tunnel Reynolds Analogy factor is reasonably close to unity, but the train value is very much less than unity, particularly at large values of train friction factor. The rate of increase of friction then exceeds that for heat transfer rate as the surface becomes more rough.

It is a fairly simple matter to make a velocity measurement in the vicinity of the moving train surface. Accordingly, the computer program has been developed further to cater for such a measurement and so to dispense with the initial assumption concerning the train friction factor. This factor is now determined as explained earlier, along with the relative roughness for the train structure. The additional computation and iteration is shown in the flow diagram, Fig 4. Instead of completion of the calculation at E, the additional steps are now included through the use of Eqs (4) and (5) as shown. The friction factors, wall stresses, and roughness are then interdependent with the velocity measurement, which only needs to be made at a single locality.

Preliminary calculations with this refined procedure have been made following the success of the basic analysis but it was

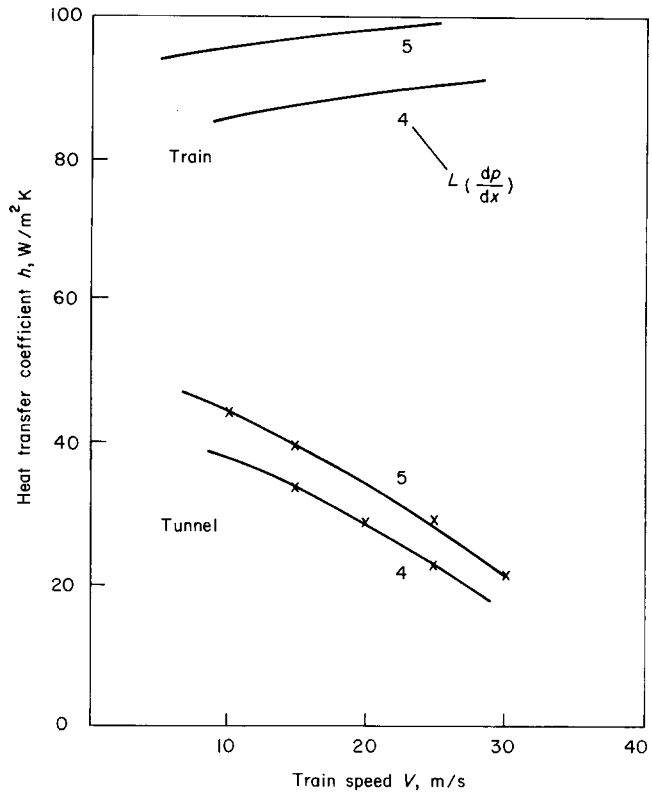


*Figure 6* Effect of train speed on train and tunnel heat transfer coefficients assuming constant pressure gradient with $f$ (train) = 0.012

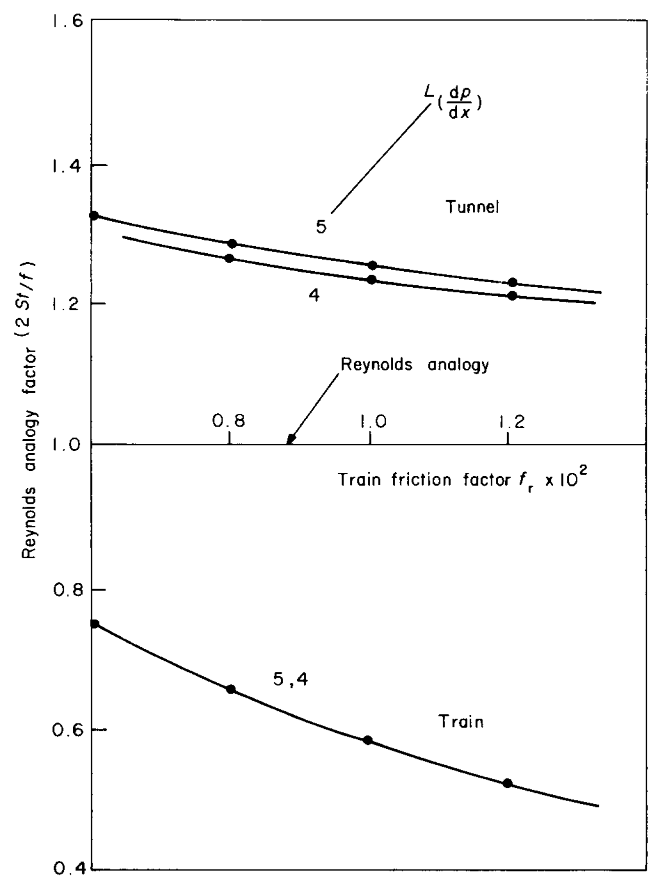


*Figure 7* Reynolds Analogy factor for train and tunnel surfaces

found that the heat transfer coefficients were not greatly affected. The correctness of the train friction factor may be readily checked with the train shear stress and the computed average velocity in the imaginary rough duct. Even with the assumed constant value of train friction factor, good consistency was obtained, which lends support to the use of the simpler procedure. Finally, the values of train heat transfer coefficient correspond closely with that given in Ref 3. It appears that the value of 90 $W/m^2 K$ given in Ref 3 is based on model experiments, so that the outcome of the present theoretical enquiry is extremely encouraging.

## Conclusions

A theoretical analysis of turbulent flow and heat transfer has been made for the situation when a train is in transit through a very long tunnel. A new model has been devised, the flows adjacent the rough and smooth surfaces being simulated by parallel-wall ducted flows for which flow and heat transfer correlations are available. An iterative computational scheme has been developed for the prediction of the flow and heat transfer parameters. There is good agreement between the flow results of the new analysis and those from a previous theoretical and experimental investigation for this case[7]. Furthermore, the predicted heat transfer coefficients for the train surface in particular are close to those determined by other workers[3]. The results as a whole are extremely meaningful and in keeping with the flow and heat transfer conditions within this complex situation of mixed surfaces with relative motion.

Accordingly, further enquiry into the technique together with development of the computational procedure is warranted.

## Acknowledgement

The authors wish to thank the British Railways Board for permission to publish this paper.

## References


1 Iguchi Yasuo. Estimation of temperature in Seikan tunnel. *Q. Rep. Railway Tech. Res. Inst. Jpn. Nat. Railways*, 1985, **26**(4), 145–150

2 Moron, P. Heat problems concerning the proposed Channel Tunnel (Problemes calorifigues soleves par la futur tunnel sous la Manche). *Rev. Gen. Chemins de Fer*, 1971, **90**, 688–695

3 Pope, C. W. and Woods, W. A. The prediction of thermal effects in railway tunnels. 5th Int. Symp. on the Aerodynamics and Ventilation of Vehicle Tunnels, Lille, France, Paper E3, 1985

4 Schlichting, H. *Boundary Layer Theory 7th Edn*, McGraw-Hill, 1979

5 Moody, L. F. Friction factors for pipe flow. *Trans. ASME*, 1944, **66**, 671–684

6 Kays, W. M. and Crawford, M. E. *Convective Heat and Mass Transfer*, Tata McGraw-Hill, 1983

7 Nayak, U. S. L., Gralewski, Z. A. and Stevens, S. J. The aerodynamic drag of tube vehicles travelling at low subsonic speeds. 2nd Int. Symp. on the Aerodynamics and Ventilation of Vehicle Tunnels, Cambridge, England, Paper E1, 1976


# Book reviews

## BASIC Programs for Steam Plant Engineers

V. Ganapathy

The book is one of a series of textbooks and reference books in mechanical engineering. It comprises 30 programs written in BASIC divided into five groups respectively treating: (1) Fuels, Combustion, and Efficiency of Boilers and Heaters; (2) Fluid Flow and Pressure Drop Calculations; (3) Heat Transfer Calculations; (4) Steam Utilisation; (5) Performance of Heat Transfer Equipment. In each case the program treats a specific calculation that is commonly required during the design or analysis of steam plant. Typical programs from each group (as numbered above) are: combustion calculations for solid and liquid fuels, sizing orifices for steam flow, estimating fin tip temperatures, steam properties after expansion and performance of economisers.

The book is well laid out and easy to use. Every program has supporting material: input, output, remarks, theory, notation for program, the program itself sometimes clarified by a flow diagram, examples (one or more) and their solution. Where needed a clarifying diagram is included. When physical properties are involved, eg, thermodynamic properties of steam, function correlations are employed. Frequently an accepted empirical expression, correlation, etc, is introduced.

This reviewer, while commending the book, has two reservations. In the first place the BASIC employed is that of IBM PC and compatible systems and thus likely to be more widley useful in the USA than elsewhere where a different dialect of BASIC may be employed. If the user has to rewrite the program the advantage otherwise gained by time-saving may be lost or diminished. Having made this point, however, it is fair to add that I selected two programs impartially and keyed them into my personal computer (Amstrad 6128). Using the data from the examples given, the program ran without needing modification giving the same answers as those shown. The second reservation is more serious. This book, published in 1986, uses Imperial Units. As readers familar with the USA will known Imperial Units are still widely used there, in some cases even by companies whose names are household words, and this despite coaxing by many of the leading engineering institutions. As S.I. Units are now used in most countries the book will surely have a restricted sale outside the USA. This is not, of course, a criticism of the book—it is an unfair disadvantage imposed upon it. If a further edition is called for the author should consider including S.I. Unit equivalents within the text. The provision of a conversions table is insufficient. The principal merit of the book—which will probably govern the number sold—lies in its clear layout and the provision of useful programs ready for immediate use. Duplicated sets of units would preserve this.

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Published price $47.50, by Marcel Dekker, 270 Madison Avenue, New York, NY 10016, USA, 168 pp.